

# Electroproduction of vector mesons: factorization of end-point contributions

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**Abstract.** The end-point contributions in the quark longitudinal momentum fraction of the virtual photon ( $\gamma^*$ ) to vector meson ( $V$ ) impact factor to the diffractive electroproduction amplitude can be factorized in terms of a generalized parton evolution of the target parton distribution. This result is used to model the helicity amplitudes  $\gamma^*p \rightarrow Vp$  in terms of small  $x$  generalized parton distributions.

## 1 Introduction

The experimental analysis of diffractive vector meson production by virtual or quasi-real photons ( $\gamma p \rightarrow Vp^*$ ) at HERA [1–10] has been accompanied by numerous theoretical and phenomenological studies, e.g. [11–23]. The main questions under discussion are the typical ones for semi-hard processes: To what extent does perturbative QCD apply? What can we learn about the non-perturbative hadronic interactions?

The factorization proof [25] gives a first answer. In the helicity amplitudes with  $\lambda_i(\gamma) = 0$  factorization holds and the contribution of a small-size  $q\bar{q}$  dipole coupled by two gluons to the exchanged pomeron dominates by power counting the contributions with additional soft exchanges. For the remaining helicity amplitudes the power counting does not result in the dominance of the short distance contribution. This is different from the related deep virtual Compton amplitude (DVCS,  $\gamma^*p \rightarrow \gamma^*p$ ), where the short distance contribution dominates in all helicity amplitudes [26].

In some presentations the situation is described by calling the dominating short distance contribution in the helicity amplitude  $\lambda_i = \lambda_f = 0$  the leading twist one and by saying that short distance factorization must not be expected for the other helicity amplitudes because of being of non-leading twist.

Owing to the DVCS case the latter argument looks unconvincing. Moreover, the amplitudes with helicities different from  $\lambda_i = \lambda_f = 0$  are accessible to experiment, e.g. in the ratio of the cross sections of longitudinal to transverse virtual photons and in the angular-decay distributions parameterized by the Schilling–Wolf ratios [1, 24]. If short distance factorization holds, the amplitude can be expressed in terms of generalized parton distributions

of the target and predictions for  $\sigma_L/\sigma_T$  [16–20] and the Schilling–Wolf ratios [21, 22] have been obtained. In some of the approaches the amplitudes have been treated in analogy to the one with  $\lambda_i = \lambda_f = 0$ , i.e. with the  $q\bar{q}$  dipole interacting via two gluon exchange coupled to the generalized gluon distribution. However, unlike the  $\lambda_i = \lambda_f = 0$  case, in the leading twist contribution to the other helicity amplitudes singularities in the momentum fraction of the quark in the dipole appear. These are related to a large transverse size of the dipole and seem to signal the factorization breakdown expected from the power counting analysis [25].

There are controversial opinions about the appropriate treatment of those end-point singularities: introducing physically motivated cut-offs, choosing damping meson wave functions or including damping quark formfactors, or relating the argument of the parton distributions to the increasing dipole size. In any case, the partial success of these perturbative approaches seem to result in a modification of the first answer to the above main questions as given by the factorization proof: The factorization breaking effects cannot be large for all helicity amplitudes.

The connection of the end-point singularities to the factorization breaking is referred to frequently. In a study [27] using the operator product language these end-point singularities appear as the obstacle to factorization.

In the present paper we end up with the opposite conclusion: The end-point contributions are factorizable; the ones appearing in the quark dipole interacting by two gluon exchange are factorized by identifying them as a leading  $\ln Q^2$  Bjorken evolution term of the two gluon ( $gg$ ) to quark–antiquark ( $q\bar{q}$ ) exchange. Our discussion relies on a specific model of the meson light-cone wave function. It includes more than the leading twist contribution in terms of distribution amplitudes. Although finally only a small part of the specific information encoded in this

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wave function enters the results on the large  $Q^2$  asymptotics the inclusion in the wave function and resummation of a geometric series of higher twist terms  $\sim \left(\frac{m_V^2}{Q^2 z \bar{z}}\right)^n$  is the essential point for understanding the physical meaning of the end-point contributions.

In Sect. 2 we start with the impact factor representation of the diffractive amplitude and specifically look to the contribution of a scattering  $q\bar{q}$  dipole coupled to the pomeron exchange by two gluons. In Sect. 3 the  $\gamma^*V$  impact factors are introduced by specifying the meson light-cone wave function. The asymptotics for large  $Q^2$  is calculated. We compare with the  $\gamma^*\gamma^*$  impact factor and continue this comparison in the following. Some details of the calculations are given in the Appendix. In Sect. 4 the logarithmic end-point contributions to the amplitudes are identified with a leading  $\ln Q^2$  contribution to the generalized Bjorken evolution. In order to make this main point clearer this identification is repeated for the  $\gamma^*p \rightarrow \gamma^*p$  amplitude with  $\lambda_i = \lambda_f = 1$ , i.e. for the well known case related to the structure function  $F_2(x, Q^2)$ . In Sect. 6 we evaluate numerically the resulting leading twist terms of the helicity amplitudes, specifying the small  $x$  parton distribution, and obtain results for quantities which can be compared with the results of the experimental analysis: the cross section ratio  $\sigma_L/\sigma_T$  and the angular-decay distribution (Schilling–Wolf) coefficients  $r_{jk}^\alpha$  in dependence on  $Q^2$  and  $t$ .

## 2 Diffractive $\gamma^*V$ amplitudes

A good starting point for analyzing high energy diffractive processes is the impact factor representation of the corresponding amplitude written in terms of partial waves:

$$\begin{aligned}
& M^{\lambda_i \lambda_f}(s, Q, q) \\
&= s \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} F^{\lambda_i \lambda_f}(\omega, Q, q) \\
&\times \left[ \left( \frac{s}{M^2(Q, m, q)} \right)^\omega + \left( \frac{-s}{M^2(Q, m, q)} \right)^\omega \right], \\
& F^{\lambda_i \lambda_f}(\omega, Q, q) \\
&= \int d^2\kappa d^2\kappa' \Phi^{\lambda_i \lambda_f}(\kappa, Q, q) \mathcal{G}(\kappa, \kappa', q, \omega) \Phi^P(\kappa', q)
\end{aligned} \tag{2.1}$$

This is a typical form obtained in perturbative analysis [29, 35]; however, it is based on more general arguments relying on the impulse approximation [30]. The field representing the exchange interaction acts on the scattering particles for a short time, much shorter than the time scale of their binding or self-interaction. The field sees just a short-time intermediate fluctuation state. The impact factors appear as the transition matrix elements ( $\gamma^* \rightarrow V$  and  $P \rightarrow P^*$  in our case) of an operator representing the action of the exchange field on the intermediate fluctuations selecting some of the fluctuations according to their interaction strength.

The diffractive exchange  $\mathcal{G}$  is called a pomeron. In phenomenology it is often substituted by a Regge pole or, according to a recent proposal [28], by two poles, a soft and a hard pomeron. In the framework of QCD we suppose the pomeron to consist of interacting gluons. In perturbative QCD this idea acquires a definite meaning in the BFKL scheme [34, 35].

In the case of hard diffraction the intermediate fluctuations is squeezed into a narrow space-time region ( $\sim \frac{1}{Q}$ ) in the vicinity of the light cone. Only short distance modes of the pomeron field can interact with this fluctuation state. In this situation both the fluctuation state and the coupling to the pomeron can be represented by perturbative QCD. Moreover, one finds that the fluctuations with a small number of partons dominates, the higher Fock states being suppressed by powers of  $\frac{1}{Q}$ . Also the exchange with a minimal number of exchange partons dominates not only by small couplings but, more important, by powers of  $\frac{1}{Q}$ . This suppression holds if the exchanged partons carry large transverse momenta  $\kappa$ ,  $Q^2 \gg \kappa^2 \gg m^2$ , and one has to make sure that contributions with extra soft exchange quanta are absent or can be absorbed into generalized parton distributions.

In the diffractive vector meson production by virtual photons the factorization in terms of a  $q\bar{q}$  intermediate fluctuation coupling to the pomeron by two gluons has been proven on a rigorous level in the case where the short distance scale is provided by the momentum squared of the virtual photon with longitudinal polarization [25].

In the case of factorization via two exchanged partons one can write

$$\mathcal{G}(k, k', q, \omega) = \frac{1}{|k|^2 |k+q|^2} \tilde{\mathcal{G}}(k, k', q, \omega). \tag{2.2}$$

In the considered case of electroproduction we have in (2.1)  $M^2(Q^2, m, q) \approx Q^2$ . We pick up the leading contribution in the  $\omega$  integral by writing

$$M^{\lambda_i \lambda_j} = \sum_p \int d^2\kappa \Phi_p(\kappa, q, Q) \frac{1}{|k|^2 |\kappa+q|^2} G'_p(x_1, x_2, q, \kappa). \tag{2.3}$$

$G'_p$  stands for the unintegrated generalized parton distribution of the proton (for recent reviews see [36–38]) resulting from the convolution of the proton impact factor  $\Phi^P$  with the projection of the exchange  $\mathcal{G}$  that couples by two exchange partons ( $p$ ) in the small Bjorken variable limit,  $x_1 = \frac{Q^2}{s}$ ,  $x_2 = \frac{m_V^2}{s}$ . The skewedness  $\xi = \frac{x_1 - x_2}{2}$  is small; we have a non-vanishing transverse momentum transfer  $q$ .

As a simplification we replace  $G'_p$  by the derivative of the gluon distribution at small  $x$

$$G'_p = \frac{\partial}{\partial \ln |\kappa|^2} [G_p(x_1, x_2, |q|^2, |\kappa|^2) T(\kappa^2, Q^2)]. \tag{2.4}$$

Here  $T(\kappa^2, Q^2)$  is the parton Sudakov formfactor; it is equal to 1 at  $\kappa^2 = Q^2$  and small for  $\kappa^2 \ll Q^2$ .

The relation to the standard notation introduced by Ji is as follows:

$$G_p(x_1, x_2) = H_p^{Ji}(x, \xi), \quad x = \frac{x_1 + x_2}{2}, \quad \xi = \frac{x_1 - x_2}{2}. \quad (2.5)$$

For the leading contribution in  $Q^2$  we expand  $\Phi_p^{\lambda_i \lambda_f}$  ( $k, q, Q^2$ ):

$$\begin{aligned} & \Phi^{\lambda_i \lambda_f}(\kappa, Q, q) \\ &= \left( \frac{\mu^2(m_V, q)}{Q^2} \right)^\tau \frac{k(k+q)^* + k^*(k+q)}{Q^2} C^{\lambda_i \lambda_f} + \dots \end{aligned} \quad (2.6)$$

The leading  $\ln Q^2$  contribution of the  $\kappa$  integral is then obtained as

$$\begin{aligned} & M^{\lambda_i \lambda_f}(s, Q, q) \\ &= \sum_p \left( \frac{\mu^2(m_V, q)}{Q^2} \right)^\tau \frac{1}{Q^2} C^{\lambda_i \lambda_f} G_p(x_1, x_2, q, Q^2). \end{aligned} \quad (2.7)$$

As an obstacle to factorization in diffractive electroproduction with transverse polarization, mentioned in the Introduction, one encounters large contributions from  $q\bar{q}$  fluctuation states with the longitudinal momentum fraction of one of the partons ( $z$  or  $\bar{z} = 1 - z$ ) small. The soft scattering quark then may couple by soft exchange gluon to the pomeron without suppression by  $1/Q$ .

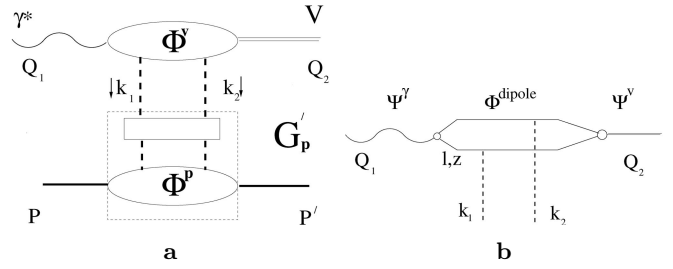
We shall investigate the expressions for impact factors with the  $q\bar{q}$  state and a model wave function  $\psi^V$  of the vector meson. We shall show that the enhanced end-point contribution at large  $Q^2$  ( $s \gg Q^2 \gg m_V^2$ ) actually arises from  $1 \gg z, \bar{z} \gg \frac{|\kappa|^2}{Q^2}$ . In this range the  $z$  integral is approximately logarithmic and this contribution can be identified as the one of the generalized Bjorken evolution [32, 33] of the two exchanged gluons  $gg$  to the exchanged quark-antiquark  $q\bar{q}$ . The latter exchange involves higher twist modes.

In this way we are going to show that the soft quark or end-point contribution can be factorized and included in the parton distribution of the diffractive exchange. The factorization of this contributions goes via a  $q\bar{q}$  exchange instead of  $gg$ . The  $gg$  coupling to the pomeron is then restricted to the contribution of the hard ( $z, \bar{z} = O(1)$ ) scattering  $q\bar{q}$  dipole.

### 3 Impact factors

The impact factor with the  $q\bar{q}$  intermediate state and two leading gluons exchange can be written as (compare Fig. 3)

$$\begin{aligned} & \Phi^{\lambda_i \lambda_f}(\kappa_1, \kappa_2) \\ &= \int d^2 \ell_1 d^2 \ell_2 dz \psi_i^{\lambda_i}(\ell_1, z) \phi^{\text{dipole}}(\ell_1, \ell_2, \kappa_1, \kappa_2) \\ & \quad \times \psi_f^{\lambda_f^*}(\ell_2 - zq, z), \\ & \phi^{\text{dipole}}(\ell_1, \ell_2, \kappa_1, \kappa_2) \\ &= \alpha_s \left[ \delta^2(\ell_2 - \ell_1) + \delta^2(\ell_2 - \ell_1 + \kappa_1 + \kappa_2) \right. \\ & \quad \left. - \delta^2(\ell_2 - \ell_1 + \kappa_1) - \delta^2(\ell_2 - \ell_1 + \kappa_2) \right]. \end{aligned} \quad (3.1)$$



**Fig. 1.** **a** Impact factor form of the  $\gamma^* P \rightarrow VP$  amplitude. The rectangular box indicates the exchange interaction and the dashed line box the unintegrated generalized gluon distribution. **b** Contribution to the  $\gamma^* V$  impact factor

The first argument in the light-cone wave functions  $\psi_{i/f}$  is the transverse momentum relative to the momentum direction of the corresponding particle. The  $\kappa_i$  are the transverse momenta components of the exchange gluons, and  $\kappa_1 + \kappa_2 = -q$  is the transverse part of the momentum transfer.

The  $\gamma^* \gamma^*$  impact factor can be treated purely perturbatively. It is obtained in the perturbative Regge asymptotics as the integral over the right-hand cut of the discontinuity of the amplitude  $\gamma^* g \rightarrow \gamma^* g$ . The four terms in  $\phi^{\text{dipole}}$  (see (3.1)) correspond to the four ways to couple the two gluons to  $q\bar{q}$ ; one of them is shown in Fig. 1b. The momentum variables are defined in the Sudakov frame:

$$\begin{aligned} Q_1 &= q' - x_1 p, \quad Q_2 = q' - x_2 p + q, \\ l &= zq' - \beta_\ell p + \ell, \\ k_i &= \alpha_i q' - \beta_i p + \kappa_i, \\ 2q' p &= s. \end{aligned} \quad (3.2)$$

The  $\delta$  functions of the mass shell condition at the right-hand discontinuity in the subenergy  $-2k_1 Q_1 \approx \beta_1 s$ , e.g., for Fig. 1b,

$$\begin{aligned} & z\delta((z - \alpha_1)(\beta_1 - \beta_\ell)s - |\ell - \kappa|^2 - m_q^2) \\ & \times \bar{z}\delta(\bar{z}(\beta_\ell - x_1)s - |\ell|^2 - m_q^2), \end{aligned} \quad (3.3)$$

are used to do the integrals over this subenergy  $\beta_1 s$  and also over the loop momentum Sudakov component  $\beta_\ell$ . The Sudakov components  $\alpha_i$  of the exchanged gluons are neglected in this step.

The two remaining propagators result in  $\Psi_i, \Psi_f$ , the light-cone wave function of  $\gamma^*$ ,

$$\begin{aligned} & \Psi(\gamma)^\lambda(\ell, z, Q) = e^{\frac{V^\lambda(\ell, z, Q)}{\left[ Q^2 + \frac{|\ell|^2 + m_q^2}{z\bar{z}} \right]}}, \\ & V^{(0)} = Q, \quad V^{(+1)} = \frac{\ell^*}{z}, \quad V^{(-1)} = \frac{\ell}{\bar{z}}, \end{aligned} \quad (3.4)$$

The sum over the fermion chiralities adds the same with  $z \leftrightarrow \bar{z}$  or amounts to the factor 2 in (3.1)

In the helicity cases  $\lambda_i = \lambda_f = \pm 1$  there is an additional chiral odd term which is included by the substitution

$$2V^{+1}(\ell_1, z)V^{*+1}(\ell_2, z) = \ell_1^* \ell_2 \left( \frac{1}{z^2} + \frac{1}{\bar{z}^2} \right) + \frac{m_q^2}{(z\bar{z})^2}. \quad (3.5)$$

As a model for the light-cone wave function of the vector meson we assume

$$\Psi^{V\lambda}(\ell, z) = f_V \frac{V^\lambda(\ell, z, m_V)}{m_V^2} \exp \left[ -\frac{|\ell|^2 + m_q^2}{z\bar{z}m_V^2} \right]. \quad (3.6)$$

The form is motivated by QCD sum rules; it is formally obtained by a Borel transformation of the propagator factor in (3.4) with respect to  $Q^2$  and by the substitution of the Borel variable by  $m_V^2$ , where  $m_V$  is of the order of the meson mass [31]. This wave function, being close to the one of  $\gamma^*$ , is a particular realization of the phenomenologically successful concept of vector dominance. Actually the explicit form of  $\Psi^V$  involves more information than necessary for the asymptotic estimate at large  $Q$ . The essential point in changing from  $\Psi^\gamma$  to  $\Psi^V$  is in removing the hard (singular in the impact parameter, the Fourier conjugate to  $\ell$ ) component, while keeping the helicity structure.

In (3.4) and (3.6) we have kept the quark mass to indicate the possible extension to the case of heavy quark vector mesons; it will be neglected in the following.

In the Appendix we consider the impact factors in some detail and calculate the leading twist contribution, for  $\gamma^*\gamma^*$  at  $Q_1^2 = x_1s$ ,  $Q_2^2 = x_2s$ , with  $x_1, x_2$  small and  $s \rightarrow \infty$  and for  $\gamma^*V$  at  $Q^2 \rightarrow \infty$ , for the representative cases of polarizations. For the asymptotic estimate we divide the range of  $z$  into  $z_0 \leq z \leq 1 - z_0 = \bar{z}_0$ ,  $0 < z < z_0$  and  $\bar{z}_0 < z < 1$ ; we have

$$\Phi = \Phi_1 + \Phi_{z_0} + \Phi_{\bar{z}_0}. \quad (3.7)$$

The results for  $z = O(1)$  are

$$\begin{aligned} \Phi_1^{\lambda_1\lambda_f}(q, k) &= \int_{z_0}^{1-z_0} \varphi_4^{\lambda_i\lambda_f}(q, k) z\bar{z}dz, \\ z\bar{z}\varphi_4^{00}(q, k) &= C_1^{00} \frac{f^{(2)}(k, q)}{\tilde{Q}^2} \\ &+ C_2^{00} \left\{ \frac{|f^{(2,**)}|^2}{\tilde{Q}^4} \frac{1}{z\bar{z}} - \left( 4 - \frac{1}{z\bar{z}} \right) \right. \\ &\times \left[ |q|^2 f^{(2)}(k, q) \right. \\ &\left. \left. + \frac{1}{2} q^{*2} f^{(2,**)}(k, q) + \frac{1}{2} q^2 f^{(2,**)}(k, q) \right] \right\}, \\ z\bar{z}\varphi_4^{01}(q, k) &= C^{01} \frac{f^{(3)}(k, q)}{\tilde{Q}^3} \left( 4 - \frac{1}{z\bar{z}} \right), \\ z\bar{z}\varphi_4^{10}(q, k) &= C^{10} \frac{f^{*(3)}(k, q)}{\tilde{Q}^3} \left( 4 - \frac{1}{z\bar{z}} \right), \\ z\bar{z}\varphi_4^{1-1}(q, k) &= C_1^{1-1} \frac{f^{(2)**}(k, q)}{\tilde{Q}^2} \\ &+ C_2^{1-1} \frac{1}{\tilde{Q}^4} \left\{ \frac{1}{2} f^{(2)}(k, q) f^{(2)**}(k, q) \frac{1}{z\bar{z}} \right. \\ &\left. + \left[ q^{*2} f^{(2)}(k, q) - |q|^2 f^{(2)**}(k, q) \right] \left( \frac{1}{z\bar{z}} - 3 \right) \right\}, \\ z\bar{z}\varphi_4^{11}(q, k) &= C^{11} \frac{f^{(2)}(k, q)}{\tilde{Q}^2} \left( \frac{1}{z\bar{z}} - 2 \right). \end{aligned} \quad (3.8)$$

We have introduced the abbreviations

$$\begin{aligned} f^{(2)}(k, q) &= k(k+q)^* + k^*(k+q), \\ f^{(2)**}(k, q) &= 2k^*(k+q)^*, \\ f^{(3)}(k, q) &= q^* f^{(2)}(k, q) + \frac{1}{2} q f^{(2)**}. \end{aligned} \quad (3.9)$$

In the case of  $\gamma^*V$  the large scale  $\tilde{Q}^2$  is just  $Q^2$ . For a smoother extrapolation back into the subasymptotic region we replace  $\tilde{Q}^2 = Q^2 + m_V^2$ . In the case  $\gamma^*\gamma^*$  the large scale  $\tilde{Q}^2$  is to be substituted in (3.8) by  $\tilde{Q}^2 = s$ ; actually  $s$  enters always multiplied by  $x_1y + x_2\bar{y}$  which we have absorbed into the coefficients  $C^{\lambda_i\lambda_f}$  in this case.

The coefficients  $C^{\lambda_i\lambda_f}$  depend on  $x_1, x_2$  in the case of  $\gamma^*\gamma^*$ :

$$\begin{aligned} C_1^{\gamma 0,0}(x_1, x_2) &= -2\sqrt{x_1x_2}Y(x_1, x_2, 1, 1, 2), \\ C_2^{\gamma 0,0}(x_1, x_2) &= 2\sqrt{x_1x_2}Y(x_1, x_2, 2, 2, 3), \\ C^{\gamma 0,1}(x_1, x_2) &= \sqrt{x_1}Y(x_1, x_2, 2, 1, 2), \\ C^{\gamma 1,0}(x_1, x_2) &= -\sqrt{2}Y(x_1, x_2, 1, 2, 2), \\ C_1^{\gamma -1,1}(x_1, x_2) &= 2Y(x_1, x_2, 1, 1, 1), \\ C_2^{\gamma -1,1}(x_1, x_2) &= -2Y(x_1, x_2, 2, 2, 2), \\ C^{\gamma 1,1}(x_1, x_2) &= x_2Y(x_1, x_2, 0, 2, 2), \end{aligned} \quad (3.10)$$

with the abbreviation

$$Y(x_1, x_2, a, b, c) = \int_0^1 \frac{dy y^a \bar{y}^b}{(x_1y + x_2\bar{y})}.$$

In the case  $\gamma^*V$  the coefficients  $C^{\lambda_i\lambda_f}$  depend on  $m_V$  and  $Q$ :

$$\begin{aligned} C_1^{V 0,0} &= C_2^{V 0,0} = -2 \frac{m_V Q}{Q^2 + m_V^2}, \\ C^{V 0,1} &= \frac{m_V^2 Q}{(m_V^2 + Q^2)^{3/2}}, \\ C^{V 1,0} &= \frac{m_V}{(m_V^2 + Q^2)^{1/2}}, \\ C_1^{V 1,-1} &= -\frac{1}{2} C_2^{1,-1} = C^{V 1,1} = \frac{m_V^2}{Q^2 + m_V^2}. \end{aligned} \quad (3.11)$$

In the asymptotically dominating amplitude  $\lambda_i = \lambda_f = 0$  we have included the next-to-leading term in the twist power expansion. This is done in order to demonstrate that the higher twist terms are accessible in the present approach and to test their effect on the numerical estimates.

The leading terms of the end-point contributions from  $z = \mathcal{O}(z_0)$  are

$$\begin{aligned} \Phi_{z_0}^{\lambda_i\lambda_f} &= C_0^{\lambda_i\lambda_f} \frac{f^{(n)}(\kappa, q)}{\tilde{Q}^n} \ln \frac{\tilde{Q}^2 z_0}{|\kappa|^2} \\ &- \frac{c^{\lambda_i\lambda_f}(k, q)}{\tilde{Q}^n} \ln \frac{|\kappa|^2}{|q|^2} + \mathcal{O}(z_0). \end{aligned} \quad (3.12)$$

The contributions from the other end point,  $\Phi_{z_0}$ , are obtained by replacing  $z_0$  by  $\bar{z}_0$ . Here  $C_0^{\lambda_i \lambda_f}$  coincide with the coefficients in front of  $\int \frac{1}{z\bar{z}}$  in  $\Phi_1^{\lambda_i \lambda_f}$  in (3.8) as expected for cancelling the auxiliary  $z_0$ .

### 4 Factorization of the end-point contributions

As a result of the previous section we have separated the end-point contributions. Starting from the impact factor with a reasonable meson wave function we have identified the end-point contributions for the kinematics  $Q^2 \gg |k|^2 \gg m^2 \sim |q|^2$  as being proportional to

$$\left( \int_{\frac{|\kappa|^2}{Q^2}}^{z_0} dz + \int_{\bar{z}_0}^{1-\frac{|\kappa|^2}{Q^2}} dz \right) \frac{1}{z\bar{z}}. \tag{4.1}$$

We observe that there is no divergence at  $z, \bar{z} \rightarrow 0$ . Spurious divergences appear only in the extrapolation of the twist expansion done for  $z = O(1)$  (see (3.8)) to  $z, \bar{z} \rightarrow 0$ . As we see, the blind extrapolation would neglect terms with powers of  $\frac{\kappa^2}{z\bar{z}Q^2}$ . The model vector meson wave function just specifies how the sum of these higher twist terms removes the end-point divergence and leads to (4.1). This point would be missed when looking only at the leading twist term of the  $q\bar{q}$  dipole scattering with two gluon coupling to the exchange.

The  $\gamma^* \gamma^*$  impact factor shows the same structure of end-point contributions at large  $s$ . The amplitude is the one of non-forward virtual Compton scattering in the Regge asymptotics  $x_1, x_2 \ll 1$ . The standard short distance factorization for the Compton amplitude at  $x_1, x_2 = O(1)$  can be continued into this region. In leading  $\ln Q^2$  we have two cases. The hard scattering subprocess may be the one of Compton scattering off a quark which enters the amplitude in convolution with the (generalized) quark distribution. This means that the coefficient functions start at tree level; the quark loop contributes only in next-to-leading order. In the other case the hard scattering subprocess is the one of  $\gamma^* g \rightarrow \gamma^* g$  via a quark loop entering the amplitude in convolution with the (generalized) gluon distribution; the corresponding coefficient function starts with the one-loop order. This suggests the way of how to treat the end-point contributions in the  $\gamma^* V$  case.

We start from the limiting form impact factor at large  $Q^2$  from which we have obtained the end-point contributions (3.12) (compare also in the Appendix (A.5) and (A.12)),

$$\begin{aligned} \Phi_{z_0}^{V \lambda_i \lambda_f} &= \frac{2}{m_V^2} \int \frac{d^2 \ell'}{z} e^{-\frac{|\ell'|^2}{zm_V^2}} \int_0^1 dz \\ &\times \left\{ W_0^{\lambda_i \lambda_f} \left( \frac{m_V^2}{Q^2 + m_V^2}, \kappa, q \right) \right. \\ &+ W_1^{\lambda_i \lambda_f} \left( \frac{m_V^2}{Q^2 + m_V^2}, \kappa, q \right) \frac{1}{Q^2} \\ &\left. + q\kappa^* + q^* \kappa + \frac{|\ell'|^2}{z} + \frac{|\kappa|^2}{z} - \dots (\kappa + q = 0) \dots \right\}. \end{aligned} \tag{4.2}$$

We have restored the integration over the loop transverse momentum  $\ell$  (see (A.4)),  $\ell' = \ell - \frac{1}{1+\lambda}(\kappa + zq) \approx \ell - \kappa$ . Further we restore the integrations over the loop momentum Sudakov components  $\beta_\ell$ , (3.2), and over the subenergy  $(q_1 - k_1)^2 \approx \beta_1 s$  by including the mass shell  $\delta$  functions (3.3). We have

$$\begin{aligned} \Phi_{z_0}^{V \lambda_i \lambda_f} &= \int_0^1 s d\beta_1 \int_0^{\beta_1} d\beta_\ell \dots z \delta(z(\beta_1 - \beta_\ell)s - |\ell'|^2) \\ &\times \delta((\beta_\ell - x_1)s - |\ell' + \kappa|^2). \end{aligned} \tag{4.3}$$

The ellipsis stands for the right-hand side of (4.2). Now the first  $\delta$  function is used to do the integral over  $z$ ; in particular we make the substitution  $\frac{|\ell'|^2}{z} = (\beta_1 - \beta_\ell)s$ . We notice that the first term in the argument of the second  $\delta$  function dominates for large  $Q^2 = x_1 s$ .

We retain only the contributions with a logarithmic contribution of the  $z$  integral. If substituted into the amplitude only the structure  $f^{(2)}(\kappa, q)$  results in a logarithmic  $\kappa$  integral allowing one to approximate the proton impact factor by the unintegrated generalized gluon distribution. In fact we pick up the leading contribution in the kinematics  $s \gg Q^2 \gg |\ell|^2 \gg |\kappa|^2 \gg |q|^2 \sim m_V^2$ . We have

$$\begin{aligned} \Phi_{z_0}^{V \lambda_i \lambda_f} &= \int_0^1 s d\beta_1 \int_0^{\beta_1} d\beta_{\ell 1} \int_{|\kappa|^2}^{Q^2} \frac{d^2 \ell'}{|\ell'|^2} \delta(\beta_{\ell 1} - x_1) \\ &\times \frac{2e^{-(\beta_1 - \beta_{\ell 1})\frac{s}{m_V^2}}}{\tilde{Q}^2 + (\beta_1 - \beta_{\ell 1})s} \left\{ \frac{W_0 |\kappa|^2}{\tilde{Q}^2 + (\beta_1 - \beta_{\ell 1})s} + W_1 \right\}. \end{aligned} \tag{4.4}$$

The resulting end-point contribution to the amplitude has the form

$$\begin{aligned} M_{z_0}^{V \lambda_i \lambda_f} &= C^{V \lambda_i \lambda_f} \frac{\bar{f}^{(n)}(q)}{\tilde{Q}^n} \int_{m_V^2}^{Q^2} \frac{d^2 \ell'}{|\ell'|^2} \alpha_S(|\ell'|^2) \\ &\times \int_0^1 s d\beta_1 \int_0^{\beta_1} d\beta_{\ell 1} \delta(\beta_{\ell 1} - x_1) \\ &\times P^V(\beta_{\ell 1}, \beta_{\ell 2}; \beta_1, \beta_2) G_g(\beta_1, \beta_2, q; |\ell'|^2), \end{aligned} \tag{4.5}$$

where

$$\bar{f}^{(n)}(q) = \int d\varphi_k \frac{f^{(n)}(\kappa, q)}{|\kappa|^2}.$$

This is just the evolution term in the integral form of the generalized GLAPD equation, corresponding to the parton splitting  $gg \rightarrow q\bar{q}$ . The splitting kernel,

$$\begin{aligned} P(\beta_{\ell 1}, \beta_{\ell 2}; \beta_1, \beta_2) & \\ &= \beta_{\ell 2}^{-1} \exp\left(-\frac{\beta_1 - \beta_{\ell 1}}{\beta_{\ell 2}}\right) \left[ 1 + \frac{\beta_1 - \beta_{\ell 1}}{\beta_{\ell 1}} \right]^{-1-a}, \end{aligned} \tag{4.6}$$

is an unconventional one, because the resulting  $q\bar{q}$  state is not the one of leading twist exchange. We have  $x_2 = \beta_{\ell 2} = \frac{m_V^2}{s} \ll 1$  and therefore the kernel results in a narrow distribution peaked around  $\beta_{\ell 1} = \beta_1$ .

## 5 Generalized Bjorken evolution

We consider in some detail the end-point contribution of the  $\gamma^*p \rightarrow \gamma^*p$  amplitude with  $\lambda_i = \lambda_f = 1$ . We expect to recover the GLAPD evolution  $gg \rightarrow q\bar{q}$  known from the case of the structure function  $F_2$  and the non-forward generalization (DVCS).

For simplicity we restrict ourselves to purely longitudinal momentum transfer,  $q = 0, x_1 \neq x_2$ . The end-point contribution to the impact factor reads

$$\begin{aligned} & \Phi^{\gamma,1,1}(k, 0)|_{z_0} \\ &= \int d^2\ell \int_0^{z_0} dz \left\{ \frac{e^2 \alpha_S \ell(\ell - \kappa)^* \frac{1}{z^2}}{\left[ x_1 s + \frac{|\ell|^2}{z} \right] \left[ x_2 s + \frac{|\ell - \kappa|^2}{s} \right]} \right. \\ & \quad \left. - \dots (\kappa = 0) \dots \right\}. \end{aligned} \quad (5.1)$$

The subtraction term is obtained from the written term by substituting  $\kappa = 0$ :

$$\begin{aligned} & \Phi^{\gamma,1,1}(k, 0)|_{z_0} \\ &= e^2 \alpha_S \int \frac{d^2\ell'}{z} \int_0^1 dy \int_0^{z_0} \frac{dz}{z} \\ & \quad \times \left\{ \frac{|\ell'|^2 - \bar{y}|\kappa|^2}{\left[ s(x_1 y + x_2 \bar{y}) + \frac{|\ell'|^2}{z} + \frac{|\kappa|^2}{z} y \bar{y} \right]^2} - \dots (\kappa = 0) \dots \right\} \\ &= e^2 \alpha_S \int \frac{d^2\ell'}{z} \int_0^1 dy \int_0^{z_0} \frac{dz}{z} \\ & \quad \times \left\{ \frac{-2 \frac{|\ell'|^2}{z} |\kappa|^2 y \bar{y} - \bar{y} |\kappa|^2 s(x_1 y + x_2 \bar{y}) + \mathcal{O}(|\kappa|^4)}{\left[ s(x_1 y + x_2 \bar{y}) + \frac{|\ell'|^2}{z} \right]^3} \right\}. \end{aligned} \quad (5.2)$$

As in (4.3) we restore the integration over  $\beta_1, \beta_\ell$  by including the mass shell  $\delta$  function. This allows one to substitute in the integrand

$$\begin{aligned} \frac{|\ell'|^2}{z} &= (\beta_1 - \beta_{l_1})s \\ &= (\beta_2 - \beta_{l_2})s, \quad \beta_{l_1} = x_1, \quad \beta_{l_2} = x_2, \end{aligned}$$

and in particular

$$s(x_1 y + x_2 \bar{y}) + \frac{|\ell'|^2}{z} = s(\beta_1 y + \beta_2 \bar{y}).$$

We obtain in analogy to the previous section

$$\begin{aligned} & \Phi^{\gamma,1,1}(k, 0)|_{z_0} \\ &= e^2 \frac{|\kappa|^2}{s} \int_{|\kappa|^2}^{z_0 Q^2} \alpha_S(|\ell'|^2) \frac{d|\ell'|^2}{|\ell'|^2} \\ & \quad \times \int d\beta_1 d\beta_2 d\beta_{l_1} \delta(x_1 - x_2 - \beta_1 + \beta_2) \delta(\beta_{l_1} - x_1) \\ & \quad \times \int_0^1 dy \frac{\beta_1 y + \beta_2 \bar{y} + 2(\beta_{l_1} - \beta_1) y \bar{y}}{[\beta_1 y + \beta_2 \bar{y}]^3} \Theta(\beta_1 - x_1). \end{aligned} \quad (5.3)$$

The result contributes to the amplitude  $\gamma^*p \rightarrow \gamma^*p$  ( $\lambda_i = \lambda_f = 1$ ) by convolution with the unintegrated gluon distribution,

$$\begin{aligned} M^{\gamma,1,1} &= e^2 \int_{m_v^2}^{z_0 Q^2} \alpha_S(|\ell|^2) \frac{d|\ell|^2}{|\ell|^2} \int_0^1 d\beta_1 d\beta_2 \\ & \quad \times \delta(x_1 - x_2 - \beta_1 + \beta_2) P(x_1, x_2; \beta_1, \beta_2) \\ & \quad \times G_g(\beta_1, \beta_2; q = 0, |\ell|^2). \end{aligned} \quad (5.4)$$

As expected, this result has the form of the evolution term in the integral representation of the GLAPD equation with the (non-forward) splitting kernel:

$$P(x_1, x_2; \beta_1, \beta_2) = \frac{\Theta(\beta_1 - x_1)}{\beta_1 \beta_2} (1 + \mathcal{O}(\beta_1 - x_1)). \quad (5.5)$$

We compare  $P(x_1, x_2; \beta_1, \beta_2)$  with the generalized GLAPD evolution kernel as calculated in [40]:

$$\begin{aligned} & H(x_1, x_2, \beta_1, \beta_2) \\ &= \frac{1}{\beta_1 \beta_2} \left\{ x_1 (J_1 - J_{11'}) + (x_1 - \beta_1) J_1 + (\beta_1 \leftrightarrow \beta_2) \right\}, \\ & J_1 = \int_{-\infty}^{\infty} \frac{d\alpha}{2\pi i} [\alpha x_1 + 1 - i\varepsilon]^{-1} [-\alpha x_2 + 1 - i\varepsilon]^{-1} \\ & \quad \times [\alpha(x_1 - \beta_1) + 1 - i\varepsilon]^{-1}, \\ & J_{11'} = \int_{-\infty}^{\infty} \frac{d\alpha}{2\pi i} [\alpha x_1 + 1 - i\varepsilon]^{-1} [\alpha(x_1 - \beta_1) + 1 - i\varepsilon]^{-1}. \end{aligned} \quad (5.6)$$

The first (second) term in the brackets corresponds to the  $gg \rightarrow qq$  transition without (with) spin flip. For the impact factor the right-hand cut discontinuity in  $\beta_1 s$  is relevant:

$$\begin{aligned} & \text{disc}_{\beta_1} H \\ &= \frac{1}{\beta_1 \beta_2} \left\{ \frac{x_1 x_2}{\beta_1 \beta_2} + \frac{(x_1 - \beta_1)(x_2 - \beta_2)}{\beta_1 \beta_2} \right\} \Theta(\beta_1 - x_1) \\ &= P(x_1, x_2; \beta_1 \beta_2). \end{aligned} \quad (5.7)$$

The familiar forward splitting kernel for  $gg \rightarrow qq$  is recovered for  $\beta_1 = \beta_2, x_1 = x_2$ .

The comparison of (5.7) and (5.5) confirms that the end-point contribution to the diffractive amplitude written in the impact factor representation is to be absorbed into the evolution of the generalized gluon distribution. Since the impact factor ansatz accounts for the Regge asymptotics the corresponding evolution contribution results merely in this approximation; in particular the subenergy  $(Q_1 - k_1)^2 \simeq s(\beta_1 - x_1)$  is small in this asymptotics.

## 6 Amplitudes and numerical estimates

The decomposition of the  $z$  range in the impact factor (3.7) leads to the result that the amplitude can be taken to be a sum of the corresponding three terms,  $M^{\lambda_i, \lambda_f} =$

$M_1^{\lambda_i, \lambda_f} + M_{z_0}^{\lambda_i, \lambda_f} + M_{\bar{z}_0}^{\lambda_i, \lambda_f}$ . According to (2.3) the contribution from  $z = \mathcal{O}(1)$  is calculated as

$$M_1^{\lambda_i, \lambda_f} = \int_{m_V^2}^{Q^2} d^2\kappa \int_{z_0}^{\bar{z}_0} z \bar{z} \varphi_4^{\lambda_i, \lambda_f}(q, \kappa) dz \frac{1}{|\kappa|^2 |q + \kappa|^2} \times G'_g(x_1, x_2, q, |\kappa|^2). \quad (6.1)$$

The leading  $\ln Q^2$  contribution of the integral over  $\kappa$  results in

$$M_1^{\lambda_i, \lambda_f} \approx \left( \int_{z_0}^{\bar{z}_0} z \bar{z} \bar{\varphi}_4^{\lambda_i, \lambda_f} dz \right) \cdot G_g(x_1, x_2, q; Q^2). \quad (6.2)$$

Here  $\bar{\varphi}_4^{\lambda_i, \lambda_f}$  are the coefficients in (3.8) accompanying  $f^{(2)}(\kappa, q)$  as in (2.6). For example, in the case  $\lambda_i = \lambda_f = 1$  we have

$$M_1^{11} \approx \left( \int_{z_0}^{\bar{z}_0} \left( \frac{1}{z \bar{z}} - 2 \right) dz \right) \frac{C^{V11}}{Q^2 + m_V^2} \cdot G_g(x_1, x_2, q; Q^2). \quad (6.3)$$

The end-point contributions are not small with  $z_0$  for the terms in (3.8) involving  $\frac{1}{z \bar{z}}$ . As explained in Sect. 4 the logarithmic  $z$  integral is a generalized Bjorken evolution (4.5) term resulting in the effective quark distribution

$$\tilde{G}_q(x_1, x_2, q, z_0 Q^2) = \int_{m_V^2}^{z_0 Q^2} \frac{d|\ell|^2}{|\ell|^2} \frac{\alpha_S(|\ell|^2)}{\alpha_S(Q^2)} G_g(x_1, x_2, q, |\ell|^2). \quad (6.4)$$

In the example  $\lambda_i = \lambda_f = 1$  this leads to

$$M_{z_0}^{11} = M_{\bar{z}_0}^{11} = \frac{C^{V11}}{Q^2 + m_V^2} \cdot \tilde{G}_q(x_1, x_2, q, z_0 Q^2). \quad (6.5)$$

The sum of the three contributions to the amplitude has the form (2.7). Notice that the effective quark distribution has been identified with the Bjorken evolution term of the gluon distribution. This entails the reasonable assumption that this distribution is small for small  $x$  at the scale  $m_V^2$ .

The generalized gluon distribution at small  $x_{1/2}$  is adopted here to be proportional to the ordinary small  $x$  gluon distribution

$$G_g(x_1, x_2, q, Q^2) = c G_g \left( \frac{x_1 + x_2}{2}, Q^2 \right) e^{-b|q|^2}. \quad (6.6)$$

For the slope parameter we adopt the value  $b = 6 \text{ GeV}^{-2}$ .

The generalized Bjorken evolution leads to a modification of this relation if it is assumed to hold at some  $Q_0^2$ . This effect has been investigated for small  $x$  and small skewedness [39] and has been applied to diffractive vector meson production [20]. In the latter paper the  $Q^2$  dependence of the cross sections is found to be changed by 50 per cent for light vector mesons at the highest  $Q^2$  ( $= 30 \text{ GeV}^2$ ). Since the aim of the numerical estimates in this paper is merely the illustration of the proposed factorization concept, we do not include the skewedness

effects here for simplicity, understanding that this would be an appropriate further improvement.

Further, for the numerical estimate we have to specify the gluon distribution. Although our approach is based on the asymptotic expansion, in  $Q^2$  for a comparison with the data we have to extrapolate the results to non-asymptotic values of this scale. Moreover, in the reconstruction of the effective quark distribution according to (6.4) we have to integrate the small  $x$  gluon distribution starting from  $m_V^2 \approx .5 \text{ GeV}^2$ . This means that we need the gluon distribution at small scales, where the standard parameterizations like MRST do not give a certain answer and where the application of the evolution equation is not reliable. Therefore we adopt the two-pomeron parameterization [28],

$$G_g(x, Q^2) \sim X_0 \left( \frac{Q^2}{1 + \frac{Q^2}{Q_0^2}} \right)^{1+\epsilon_0} \left( 1 + \frac{Q^2}{Q_0^2} \right)^{\epsilon_0/2} x^{-\epsilon_0-1} + X_1 \left( 1 + \frac{Q^2}{Q_1^2} \right)^{\epsilon_1/2} x^{-\epsilon_1-1},$$

where  $\epsilon_0 = 0.43$ ,  $\epsilon_1 = 0.08$ ,  $X_0 = 0.0014$ ,  $X_1 = 0.5954$ ,  $Q_0^2 = 9.108$ ,  $Q_1^2 = 0.5894$ .

Again, we have no particular reason to favor this parameterization besides of its convenience for use as an illustration. We did not try to optimize the results with regard to different possible input parameterizations.

The cut-off  $z_0$  is to separate the  $z$  range of order unity from the end-point regions; a value  $z_0 = 0.1, \dots, 0.2$  is reasonable and indeed in this range the  $z_0$  dependence of the result is weak. The condition that  $z_0 Q^2$  is much larger than  $m_V^2$  leads to the restriction of the applicability of the estimates to  $Q^2 > 5 \text{ GeV}^2$ .

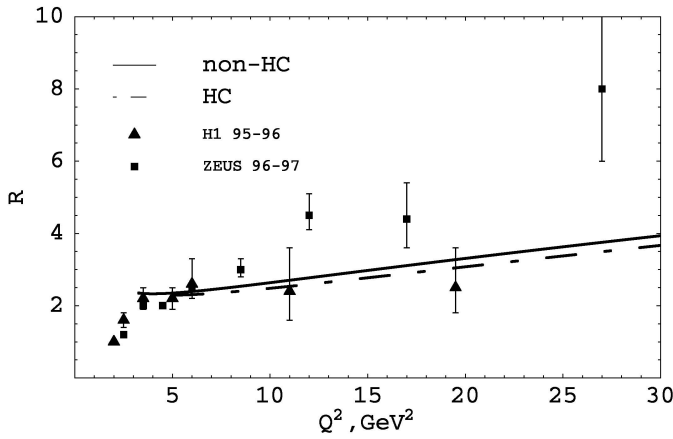
We have calculated in the given approximation with the input specified above the  $Q^2$  dependence of the ratio of longitudinal to transverse virtual photon diffractive cross sections. We do this including all helicity amplitudes ( $R$ ) and also with the helicity conserving amplitudes only ( $R_0$ ), the latter case corresponds to the ratio usually obtained in the data analysis. Thus we calculate

$$R(Q^2) = \frac{\sigma_L(Q^2)}{\sigma_T(Q^2)}, \quad R_0(Q^2) = \frac{\sigma_L^{(0)}(Q^2)}{\sigma_T^{(0)}(Q^2)}, \quad (6.7)$$

where

$$\begin{aligned} \sigma_L(Q^2) &= \int dt (|M^{00}|^2 + 2|M^{01}|^2), \\ \sigma_L^{(0)}(Q^2) &= \int dt |M^{00}|^2, \\ \sigma_T(Q^2) &= \int dt (|M^{11}|^2 + |M^{10}|^2 + |M^{1-1}|^2), \\ \sigma_T^{(0)}(Q^2) &= \int dt |M^{11}|^2. \end{aligned} \quad (6.8)$$

The results are shown in Fig. 2 together with HERA data.  $R_0$  deviates from  $R$  by 10 per cent at higher  $Q^2$ . The inclusion of the next-to-leading terms in the twist expansion does not lead to noticeable changes.



**Fig. 2.** Ratio of the longitudinal and transverse elastic  $\rho^0$  electroproduction cross sections as a function of  $Q^2$ . The dotted line corresponds to the assumption of helicity conservation, the solid line takes into account spin flip amplitudes

The coefficients in the angular-decay distribution are more sensitive to the helicity dependence, because in some of them the small flip amplitudes enter in the first power. We use the following expressions for the coefficients  $r_{ik}^\alpha$  in terms of the helicity amplitudes  $M^{\lambda_i \lambda_f}$  simplified in comparison to [24] for the appropriate case of the virtual photon polarization parameters being  $\epsilon \approx 1, \delta \approx 0$ . We have

$$r_{00}^{04} \propto \frac{1}{N} (|M^{00}|^2 + |M^{10}|^2),$$

$$r_{00}^5 \propto \frac{1}{N} \text{Re}(M^{00*} M^{10}),$$

$$r_{11}^5 \propto (\text{Re}(M^{01} M^{*11}) - \text{Re}(M^{01} M^{*1-1})),$$

$$r_{00}^1 \propto -\frac{1}{N} |M^{10}|^2,$$

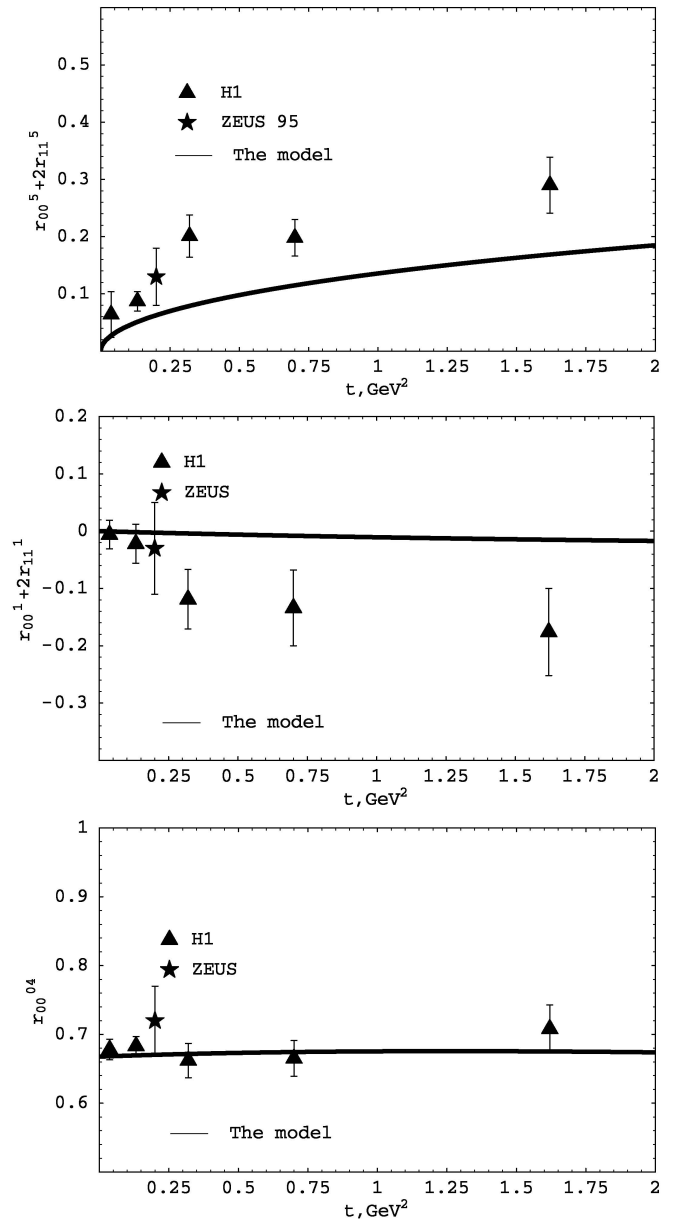
$$r_{11}^1 \propto \frac{1}{N} (M^{1-1} M^{*11} + M^{11} M^{*1-1}),$$

$$N = |M^{00}|^2 + |M^{10}|^2 + 2|M^{01}|^2 + |M^{11}|^2 + |M^{1-1}|^2.$$

Results are given for  $r_{00}^{04}, r_{00}^1 + 2r_{11}^1, r_{00}^5 + 2r_{11}^5$ . In the data analysis the first coefficient is extracted from the dependence on the polar angle of the  $\pi^+$  in the vector meson rest frame with respect to the vector meson momentum direction. Actually  $R_0$  is obtained from this coefficient by a formula valid for the ratio in the approximation of helicity conservation. The other combinations are the ones obtained in the data analysis from the dependence on the angle between the lepton scattering and vector meson production planes.

The  $t$  dependence of the coefficients  $r_{ij}^\alpha$  has been calculated at a fixed value of  $Q^2 = 10 \text{ GeV}^2$ . The results are shown in Fig. 3 in comparison with the data for the broad range  $Q^2 = 3, \dots, 30 \text{ GeV}^2$  with the average values of  $6.5 \text{ GeV}^2$  for the ZEUS data points and  $5 \text{ GeV}^2$  for the H1 data points.

In evaluating the  $Q^2$  dependence we have substituted in the amplitudes the average value of  $t$  determined by the slope parameter  $b$ . The results are shown in Fig. 4.



**Fig. 3.**  $r_{00}^5 + 2r_{11}^5, r_{00}^1 + 2r_{11}^1, r_{00}^{04}$  as a function of transferred momentum  $t$

## 7 Discussion

The diffractive  $\gamma^* V$  amplitudes constructed with a  $q\bar{q}$  dipole impact factor involve enhanced contributions from the end-point regions in the quark longitudinal momentum fraction  $z$ , besides the leading contribution to the  $\lambda_i = \lambda_f = 0$  helicity amplitude. These end-point contributions appear as singularities in the extrapolation of the twist expansion. However, resummed corrections proportional to  $\left(\frac{m_V^2}{z\bar{z}Q^2}\right)^n$  remove the singularities at  $z, \bar{z} = 0$  and result in logarithmically enhanced end-point contributions. The latter are shown to be the ones of the generalized Bjorken evolution of the  $t$ -channel parton exchange  $gg \rightarrow q\bar{q}$ . The short distance factorization of the diffrac-



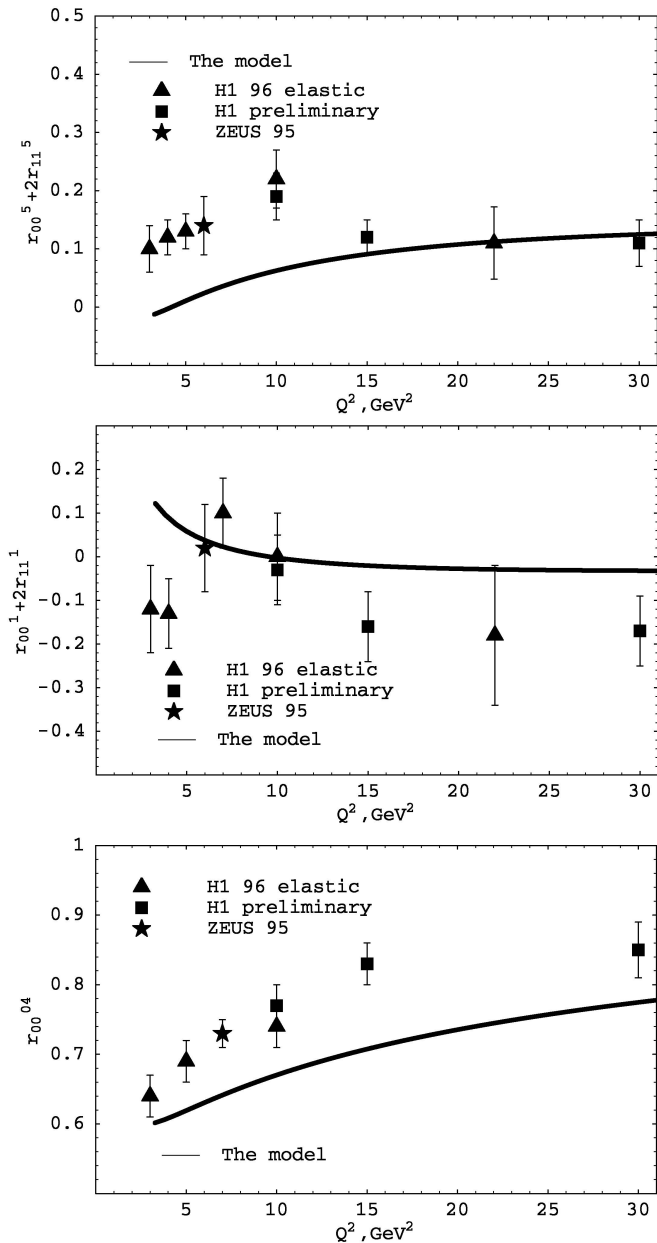


Fig. 4.  $r_{00}^5 + 2r_{11}^5$ ,  $r_{00}^1 + 2r_{11}^1$ ,  $r_{00}^4$  as a function of  $Q^2$

tive amplitude in terms of the  $\gamma^*V$  transition and the pomeron exchange therefore involves contributions of two types: one with a scattering  $q\bar{q}$  dipole coupled by  $gg$  to the pomeron and one with a (Compton-like) scattering  $q$  or  $\bar{q}$  coupled by  $q\bar{q}$  to the pomeron. The  $q\bar{q}$  exchange differs from the leading twist one.

We have chosen the impact factor form of the diffractive amplitude as a starting point. Our lack of understanding of the proton impact factor and of the pomeron coupling to it has been managed by replacing their convolution by the unintegrated generalized parton distribution, a standard step in  $k_T$  factorization schemes. This is justified in so far as it results at large  $Q^2$  in the parton distribution convoluted with the coefficient function resulting

from the  $\gamma^*V$  impact factor. Notice that in the leading  $\ln Q^2$  approximation, contributions of the impact factors, depending on the azimuthal angle of the exchanged parton momentum,  $\frac{\kappa}{|\kappa|}$ , drop out. We see here a possible source of corrections, which may be relevant at moderate  $Q^2$ .

The resulting large  $Q^2$  asymptotics of the amplitudes consists of terms with the generalized small  $x$  gluon distribution and with the effective  $q\bar{q}$  distribution. The latter appears as an additional non-perturbative input. However, its main contribution results from the evolution of the gluon distribution with a splitting function transferring the longitudinal momentum fraction from  $g$  to  $q$  almost unchanged.

In this way we see that the construction of the diffractive amplitude with a scale of the gluon distribution replaced by  $z\bar{z}Q^2$  [19] is actually an approximation to the factorization proposed here, because with that scale replacement the end-point contributions of the  $z$ -integration are approximately the  $q\bar{q}$  exchange contribution by  $gg \rightarrow q\bar{q}$  evolution. This scale replacement has been applied in a previous study of the polarization effects [21].

In our construction the terms  $\sim \left(\frac{m_V^2}{z\bar{z}Q^2}\right)^n$  improving the end-point behavior are introduced via the vector meson light-cone wave function. It is modelled starting from the perturbative  $\gamma^*$  wave function by keeping its helicity structure and removing its hard contributions. Wave functions sharing these basic features lead to similar results for the large  $Q^2$  helicity amplitudes. In the present scheme the higher twist correction to the amplitudes can be calculated. The leading asymptotics involves the value of the impact parameter wave function at the origin and this can be recast in of the distribution amplitude formulation.

We did not include the hard scale evolution of the wave function [33]. In the numerical estimates we did not include the skewedness effects. We have used a particular parameterization of the small  $x$  gluon distribution covering the small  $Q^2$  range needed in particular for reconstructing the effective  $q\bar{q}$  distribution therefrom.

The results for the cross section ratio and the Schilling–Wolf coefficients have been calculated using this input with no attempts of fitting. We have shown that they describe the basic features of the data.

Including the mentioned improvements the proposed factorization scheme provides a theoretical framework applicable to diffractive electroproduction free of previous uncertainties about the end-point contributions and therefore useful for extracting information about the structure of the hadrons involved from data analysis. We expect that this scheme applies to other hard diffractive processes.

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**Table 1.** List of the numerator  $\langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' (y, Q_1, Q_2)$  for the representative cases of the helicities

$\lambda_i$	$\lambda_f$	$\langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' (y, Q_1, Q_2)$
0	0	$2Q_1 Q_2$
0	-1	$-Q_1 y (\kappa + zq) \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)$
1	0	$Q_2 \bar{y} (\kappa + zq)^* \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)$
1	-1	$y \bar{y} (\kappa + zq)^* \frac{2}{z \bar{z}}$
1	1	$- [\kappa + zq]^2 \bar{y} + z \bar{z} Q_1^2 \left( \frac{1}{z^2} + \frac{1}{\bar{z}^2} \right)$

## Appendix

We take the simple form of (3.1) into account and write

$$\begin{aligned} \Phi(k, q) &= \int_0^1 dz z \bar{z} [\varphi(k, z, q) + \varphi(-k - q, z, q) \\ &\quad - \varphi(0, z, q) - \varphi(-q, z, q)], \\ \varphi(k, z, q) &= \int d^2 \ell_1 d^2 \ell_2 \delta(\ell_2 - \ell_1 - \kappa - zq) \\ &\quad \times \Psi_i^{\lambda_i}(\ell_1) \Psi_f^{*\lambda_f}(\ell_2). \end{aligned} \quad (\text{A.1})$$

For the  $\gamma^* \gamma^*$  case we have

$$\varphi^\gamma(\kappa, q, z) = \int d^2 \ell \frac{e^2 \alpha_S V^{\lambda_i} V^{\lambda_f *}}{\left[ Q_1^2 + \frac{|\ell|^2 + m_q^2}{z \bar{z}} \right] \left[ Q_2^2 + \frac{|\ell - \kappa - zq|^2 + m_q^2}{z \bar{z}} \right]}. \quad (\text{A.2})$$

Doing the integration over the transverse momentum we obtain

$$\varphi^\gamma(\kappa, z, q) = e^2 \alpha_S \pi \int_0^1 \frac{dy}{y \bar{y}} \frac{\langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' (y, Q_1, Q_2)}{\left[ \frac{Q_1^2}{y} + \frac{Q_2^2}{y} + \frac{m_q^2}{z \bar{z} y \bar{y}} + \frac{|\kappa + zq|^2}{z \bar{z}} \right]}. \quad (\text{A.3})$$

For the  $\gamma^* V$  case we have

$$\begin{aligned} \varphi^V(\kappa, q, z) &= \int d^2 \ell_2 \exp \left( - \frac{|\ell_2|^2 + m_q^2}{m_V^2 z \bar{z}} \right) \\ &\quad \times \frac{ef_V \alpha_S V^{\lambda_i} V^{\lambda_f *}}{\left[ Q^2 + \frac{|\ell_2 - \kappa - zq|^2 + m_q^2}{z \bar{z}} \right] m_V^2}. \end{aligned} \quad (\text{A.4})$$

The result of the integration over  $\ell_2$  can be written as

$$\begin{aligned} \varphi^V(\kappa, z, q) &= ef_V \alpha_S \pi \int_0^1 \frac{d\lambda}{1 + \lambda} \frac{\langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' \left( \frac{\lambda}{1 + \lambda}, Q_1, m_V \right)}{m_V^2} \\ &\quad \times \exp \left[ - \frac{Q^2}{m_V^2} \lambda - \frac{m_q^2 (1 + \lambda)}{z \bar{z} m_V^2} - \frac{|\kappa + zq|^2}{m_V^2 z \bar{z}} \frac{\lambda}{1 + \lambda} \right]. \end{aligned} \quad (\text{A.5})$$

We list the numerator  $\langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' (y, Q_1, Q_2)$  for the representative cases of the helicities in Table 1.

As expected, the resulting expressions obey

$$\varphi(\kappa, z, q) = \varphi(-\kappa - q, \bar{z}, q), \quad (\text{A.6})$$

and in the integral (A.1) the number of terms can be reduced to two.

We calculate the asymptotics of the integral at first for  $z = \mathcal{O}(1)$  for the  $\gamma^* \gamma^*$  case in the region  $s \rightarrow \infty$ ,  $Q_i^2 = s x_i$ , with fixed  $x_i \ll 1$ ,

$$\begin{aligned} \varphi^\gamma(k, z, q) &= e^2 \alpha_S \pi \int_0^1 dy \langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' (y, s x_1, s x_2) \\ &\quad \times \left\{ \frac{1}{s} (x_1 y + x_2 \bar{y})^{-1} - \frac{1}{s^2} \frac{|\kappa + zq|^2 y \bar{y}}{(x_1 y + x_2 \bar{y})^2 z \bar{z}} + \mathcal{O} \left( \frac{1}{s^3} \right) \right\}, \end{aligned} \quad (\text{A.7})$$

and for the case  $\gamma^* V$  in the region  $Q^2 \rightarrow \infty$ ,

$$\begin{aligned} \varphi^V(\kappa, z, q) &\approx ef_V \alpha_S \pi \langle V_i^{\lambda_i} V_f^{*\lambda_f} \rangle' \left( \frac{m_V^2}{Q^2 + m_V^2}, Q_1, m_V \right) \\ &\quad \times \left\{ \frac{1}{Q^2 + m_V^2} - \frac{|\kappa + zq|^2}{(Q^2 + m_V^2) z \bar{z}} + \mathcal{O} \left( \frac{1}{(Q^2 + m_V^2)^3} \right) \right\}. \end{aligned} \quad (\text{A.8})$$

Notice that the numerator includes terms proportional to  $s$  or  $Q^2$  if  $\lambda_i = \lambda_f$ . A further term in the expansion has to be included, leading to cancellation if  $\lambda_i = \lambda_f = \pm 1$ . In the case  $\gamma^* V$  it is appropriate to choose the expansion parameter as  $\frac{m_V^2}{Q^2 + m_V^2}$ , resulting in an improvement of the extrapolation in the region of moderate  $Q^2 > m_V^2$ .

We obtain in the case  $\gamma^* \gamma^*$

$$\begin{aligned} \varphi^{\gamma,00}(\kappa, z, q) z \bar{z} &= C_1^{\gamma,00}(x_1, x_2) \frac{|\kappa + zq|^2}{s} + C_2^{\gamma,00}(x_1, x_2) \frac{|\kappa + zq|^4}{s^2} \frac{1}{z \bar{z}} \\ &\quad + \mathcal{O} \left( \frac{1}{s^3} \right), \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} \varphi^{\gamma,01}(\kappa, z, q) z \bar{z} &= C_1^{\gamma,01}(x_1, x_2) \frac{|\kappa + zq|^2 (k + zq)}{s^{3/2}} \left( \frac{1}{z} - \frac{1}{\bar{z}} \right) + \mathcal{O} \left( \frac{1}{s^3} \right), \end{aligned}$$

$$\begin{aligned} \varphi^{\gamma,10}(\kappa, z, q) z \bar{z} &= C_1^{\gamma,10}(x_1, x_2) \frac{|\kappa + zq|^2 (\kappa + zq)^*}{s^{3/2}} \left( \frac{1}{z} - \frac{1}{\bar{z}} \right) + \mathcal{O} \left( \frac{1}{s^3} \right), \end{aligned}$$

$$\begin{aligned} \varphi^{\gamma,1-1}(\kappa, z, q) z \bar{z} &= C_1^{\gamma,1-1}(x_1, x_2) \frac{(k + zq)^{*2}}{s} \\ &\quad + C_2^{\gamma,1-1}(x_1, x_2) \frac{(k + zq)^{*2} |\kappa + zq|^2}{s^2} \frac{1}{z \bar{z}} + \mathcal{O} \left( \frac{1}{s^3} \right), \end{aligned}$$

$$\begin{aligned} \varphi^{\gamma,11}(\kappa, z, q) z \bar{z} &= C_1^{\gamma,11}(x_1, x_2) \frac{|\kappa + zq|^2}{s} z \bar{z} \left( \frac{1}{z^2} + \frac{1}{\bar{z}^2} \right) + \mathcal{O} \left( \frac{1}{s^3} \right). \end{aligned}$$

We use the abbreviation for  $C^\gamma$  of (3.10). In the case  $\gamma^* V$  we have similar formulae.

**Table 2.** Expressions for calculation of the integrand of (A.1)

$\varphi(\kappa, z, q)$	$\varphi_4(\kappa, z, q)$
$ \kappa + zq ^2$	$\kappa(\kappa + q)^* + \kappa^*(\kappa + q) = f^{(2)}(\kappa, q)$
$(\kappa + zq) \kappa + zq ^2$	$(z - \bar{z})(f^{(2)}(\kappa, q) + q^*\kappa(\kappa + q)) = (z - \bar{z})f^{(3)}(\kappa, q)$
$(\kappa + zq)^{*2}$	$2\kappa^*(\kappa + q)^* = f^{(2,**)}(\kappa, q)$
$(\kappa + zq)^{*2} \kappa + zq ^2$	$\frac{1}{2}f^{(2)}f^{(2,**)} + (1 - 3z\bar{z})[q^{*2}f^{(2)} +  q ^2f^{(2,**)}]$
$ \kappa + zq ^4$	$\frac{1}{4} f^{(2,**)} ^2 + (1 - 4z\bar{z})\left[ q ^2f^{(2)} + \frac{1}{2}q^{*2}f^{(2,**)*} + \frac{1}{2}q^2f^{(2,**)}\right]$

When calculating the integrand of (A.1),

$$\varphi_4 = \varphi(\kappa, z, q) + \varphi(-\kappa - q, z, q) - \varphi(0, z, q) - \varphi(-q, z, q), \tag{A.10}$$

the dependence on  $z$  and  $\kappa, \kappa + q$  disentangles. We list the relevant expressions in Table 2.

In this way we obtain the twist expansion valid for  $z = O(1)$ . The result for the case  $\gamma^*V$  is given in (3.8); the one for  $\gamma^*\gamma^*$  can be recovered by substituting  $\frac{f^{(n)}}{(Q^2+m_V^2)^{n/2}}$  by  $\frac{f^{(n)}}{s^{n/2}}$  and  $C^{V,\lambda_i,\lambda_f}$  by  $C^{\gamma,\lambda_i,\lambda_f}$ .

We now calculate the end-point contribution to the asymptotic expansion in  $s$  for the  $\gamma^*\gamma^*$  case:

$$\begin{aligned} & \int_0^{z_0} z\bar{z}\varphi^\gamma(\kappa, z, q)dz \\ &= \int_0^1 dy \int_0^{z_0} dz \\ & \times \frac{W_0(y, \kappa) + W_1(y, \kappa)\frac{1}{z}}{\left[s(x_1y + x_2\bar{y}) + y\bar{y}(\kappa q^* + \kappa^*q + |\kappa|^2) + \frac{|\kappa|^2 y\bar{y}}{z}\right]} \\ & + \mathcal{O}(z_0), \end{aligned} \tag{A.11}$$

and in  $Q^2$  for the  $\gamma^*V$  case:

$$\begin{aligned} & \int_0^{z_0} z\bar{z}\varphi^V(\kappa, z, q)dz \tag{A.12} \\ &= \int_0^{z_0} dz \frac{W_0\left(\frac{m_V^2}{Q^2+m^2}, \kappa\right) + W_1\left(\frac{m_V^2}{Q^2+m^2}, \kappa\right)\frac{1}{z}}{\left[Q^2 + m_V^2 + (\kappa q^* + \kappa^*q + |\kappa|^2) + \frac{|\kappa|^2}{z}\right]} + \mathcal{O}(z_0). \end{aligned}$$

The numerator results from the expansion of  $z\bar{z}\langle V_i V_f \rangle'$  for small  $z$ .  $W_1$  is non-vanishing only in the case  $\lambda_i = \lambda_f = \pm 1$ . In the case  $\lambda_i = \lambda_f = 0$  both  $W_0$  and  $W_1$  vanish, here the small  $z$  expansion in the numerator starts with  $W_{-1} \sim z$ .

Thus we are lead to calculate the integrals

$$\begin{aligned} I_a(\kappa, Q^2) &= \int_0^{z_0} dz \frac{z^{-a}}{\left[Q^2 + \frac{|\kappa|^2}{z}\right]} \tag{A.13} \\ &= \left(-\frac{|\kappa|^2}{Q^2}\right)^{-a+1} Q^{-2} \ln \frac{Q^2 z_0 + |\kappa|^2}{|\kappa|^2} + \mathcal{O}(z_0) \end{aligned}$$

and obtain

$$\begin{aligned} & \int_0^{z_0} z\bar{z}\varphi^V(\kappa, z, q)dz \\ &= W_1(y, \kappa)I_1(\kappa, \tilde{Q}^2) + W_0(y, \kappa)I_0(\kappa, \tilde{Q}^2) + \mathcal{O}(z_0). \end{aligned}$$

In the case  $\gamma^*\gamma^*$  we have to substitute  $\tilde{Q}^2$  by  $s(x_1y+x_2\bar{y})+y\bar{y}(\kappa q^* + \kappa^*q)$ ; in the vector meson case we substitute  $y$  by  $\frac{m_V^2}{Q^2+m_V^2}$  and  $\tilde{Q}^2$  by  $Q^2 + m_V^2 + \kappa^*q + \kappa q^*$ . The small term  $(\kappa^*q + \kappa q^*)$  matters in the case  $\lambda_i = \lambda_f = \pm 1$  only. For illustration it is enough to do one case,  $\gamma^*V, \lambda_i = 0, \lambda_f = 1$ , explicitly:  $W_1 = 0, W_0 = -yQ\kappa$ ,

$$\begin{aligned} \Phi^{V,01}|_{z_0} &= \int_0^{z_0} z\bar{z}\varphi_4^{V,01}(\kappa, z, q)dz \\ &\approx \pi \frac{m_V^2 Q}{(m_V^2 + Q^2)} \left\{ \frac{|\kappa|^2}{Q^4} \ln \frac{Q^2 z_0 + |\kappa|^2}{|\kappa|^2} \right. \\ & \quad - \frac{(\kappa + q)|\kappa + q|^2}{Q^4} \ln \frac{Q^2 z_0 + |\kappa + q|^2}{|\kappa + q|^2} \\ & \quad \left. - \frac{q|q|^2}{Q^4} \ln \frac{Q^2 z_0 + |q|^2}{|q|^2} \right\}. \end{aligned} \tag{A.14}$$

In the kinematics  $z_0 Q^2 \gg |\kappa|^2 \gg |q|^2$  this can be approximated by

$$\Phi^{V,01}|_{z_0} \approx \pi \frac{m_V^2 Q}{(m_V^2 + Q^2)^{3/2}} \left\{ \frac{f^{(3)}(\kappa, q)}{(m_V^2 + Q^2)^{3/2}} \ln \frac{Q^2 z_0}{|\kappa|^2} \right\}. \tag{A.15}$$

In this way we obtain (3.12) confirming the matching with the expression (3.8) for  $z = O(1)$ , i.e. the cancellation of the auxiliary  $z_0$ .

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